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Modeling of COVID-19 Transmission Dynamics Extended with a Comprehensive Mitigation Protocol to Predict Health, Cost and Productivity Outcomes

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***Abstract*—This paper reports on the development of a model of COVID-19 transmission dynamics that takes into account a comprehensive mitigation protocol to better inform decision makers on COVID-19 response. The comprehensive mitigation protocol includes (1) personal protection and social distancing, (2) use of smart applications for symptom reporting and contact tracing, (3) targeted testing based on identification of individuals with possible exposure and/or infection via symptom reporting and contact tracing, (4) surveillance testing, and (5) shelter, quarantine and isolation procedures. The proposed model (1) extends a common epidemiological discrete dynamic model with the comprehensive mitigation protocol, (2) uses Bayesian probability analysis to estimate the conditional probabilities of being in non-circulating epidemiological sub-compartments as a function of the mitigation protocol parameters, based on which it (3) estimates transition ratios among the compartments, and (4) computes a range of key performance indicators including health outcomes, mitigation cost and productivity loss. The proposed model can serve as a critical component for COVID-19 mitigation recommender systems, as part of a broader effort to support urgent pandemic response.**

***Index Terms*—COVID-19, coronavirus, SARS-CoV-2, pandemic, mitigation protocol, epidemiological modeling, mathematical modeling, compartmental model, intervention strategies**

I. Introduction

In March 2020, the World Health Organization (WHO) declared the COVID-19 outbreak a pandemic, because of its rapid spread across the world [2], [10]. The total number of COVID-19 cases exceeded 28M, and the death toll exceeded 500,000 in the US alone, as of beginning of March, 2021 [4]. The COVID-19 pandemic has impacted almost every aspect of life, forcing individuals, communities, and institutions to rapidly shift to a new normal [7], [9], [13]. This paper deals with predictive modeling of COVID-19 dynamics and understanding causal effects of mitigation strategies, as a way to help develop and implement strategic response efforts to the pandemic [16].

Susceptible, Exposed, Infected, Recovered (SEIR) compartmental models are most commonly used in epidemiology to understand infectious disease dynamics [1], [6], [8], [15], which are based upon the basic Susceptible, Infected, Recovered compartmental models [3], [5]. Recently, the authors of [11] extend the standard SEIR compartmental model to assess social distancing mitigation on COVID-19 dynamics using factors specific to COVID-19, resulting in the Susceptible, Unsusceptible, Exposed, Infected, Hospitalized, Critical, Dead, Recovered (SUEIHCDR) dynamic model, described as a system of differential equations. The work [14] makes recommendations on how to re-open a university campus based on a variant of the SEIR model, extended with an isolation pool from varying frequency of asymptomatic testing of the university population. However, to the best of our knowledge, there are no prior models that take into account a comprehensive parameterized protocol of interrelated mitigation strategies, including social distancing and use of personal protection equipment (PPE), the use of enhanced contact tracing (ECT) and symptoms reporting (SR) applications (apps) [12], frequency of surveillance testing, and number of tests requested for those marked for quarantine/isolation by the ECT and SR applications. Understanding the causal effects of a comprehensive mitigation protocol is critical for decision making to produce actionable recommendations on COVID-19 mitigation. This is exactly the focus of this paper.

More specifically, the key contribution of this paper is the development of the COVID-19 epidemiological model extended with a comprehensive mitigation protocol, so that it could be used to recommend Pareto-optimal mitigation strategies. We adapted the Susceptible, Unsusceptible, Exposed, Infected, Hospitalized, Critical, Dead, Recovered (SUEIHCDR) model of COVID-19 from [11], by extending the first four categories with Non-Circulating (shelter, quarantine or isolation) and Circulating sub-compartments. Critically, we also extend the epidemiological model with a comprehensive mitigation protocol, with parameters including (1) personal protection and social distancing mitigation ratios, (2) population ratios with smart apps for symptoms reporting and contact tracing, (3) the number of tests per individual marked by each of the apps, (4) the ratio of marked (by ECT and/or SR apps) individuals that are requested to stay in non-circulation despite having a negative-test, and (5) the testing frequency of asymptomatic individuals on a random, round-robin basis. Technically, the model (1) uses Bayesian probability analysis to estimate the

conditional probabilities of being in Non-Circulating sub-compartments as a function of mitigation protocol parameters and (2) computes transition ratios among the compartments as part of a discrete dynamic model. The model also computes Key Performance Indicators (KPIs) including (1) health outcomes, in terms of all compartments, both aggregated and over the time horizon; (2) the mitigation cost and its breakdown, and; (3) productivity loss in terms of percentage of Non-Circulating population. The formal model is also implemented in Python.

The paper is organized as follows. Section II overviews the reference SUEIHCDR model from [11], our extension to Non-Circulating (NC) compartments and the proposed parameterized mitigation protocol. Section III describes the model input and output by example. Section IV presents a formal mathematical model. Finally, Section V concludes and briefly outlines some open research questions.

II. Reference Model, Non-Circulating Compartments and Mitigation Protocol

A. Reference SUEIHCDR Model

The authors of [11] extended a generalized Susceptible, Exposed, Infected, Recovered (SEIR) compartmental model by using factors specific to COVID-19 to investigate the COVID-19 pandemic in the US. The extended model comprises of eight compartments: Susceptible, Unsusceptible, Exposed, Infected, Hospitalized, Critical, Dead, and Recovered (SUEIHCDR). Susceptible individuals may become Unsusceptible, or Exposed (through close contact with Infected individuals). Exposed in [11] means that an individual is in the incubation period and not yet infectious, until they move into the Infected compartment (and becomes infectious). In turn, Infected individuals move into the Recovered compartment, or to Hospitalized compartment. From Hospitalized, individuals either become Recovered, or move into Critical compartment, from which they can move back to Hospitalized, or to Dead compartment. The model in [11] is (continuous) dynamic, described using a system of differential equations. However, the model in [11] lacks a comprehensive mitigation protocol and its causal effect on the progression of COVID-19 over time. We propose and describe these extensions in the next sections.

B. Non-Circulating Compartments and Parameterized Mitigation Protocol

As depicted in Figure 1, we extend the base compartments (Unsusceptible, Susceptible, Exposed, and Infected) $\{U, S, E, I\}$ with Circulating and Non-Circulating sub-compartments as follows. UNC and UC denote Non-Circulating (sheltered) and Circulating sub-compartments of U; SNC and SC denote Non-Circulating (sheltered) and Circulating sub-compartments of S; ENC and EC denote Non-Circulating (quarantined) and Circulating sub-compartment of E; INC and IC denote Non-Circulating (isolated) and Circulating sub-compartments of I. We denote by $BCsub = \{UC, UNC, SC, SNC, EC, ENC, IC, INC\}$ the set of all sub-compartments. As we discuss in the extended model section, these sub-compartment affect the transitions among the base compartments.

Furthermore, we extend the epidemiological model with a comprehensive mitigation protocol parameterized with (1) the personal protection and social distancing mitigation ratio, (2) the population ratio with smart apps for symptoms reporting and contact tracing, (3) the number of tests per individual marked by each of the applications, (4) the population ratio of (randomly selected) individuals marked by apps but negatively-tested individuals to be put in non-circulation due to low test sensitivity, and (5) the testing frequency of asymptomatic individuals on a random, round-robin basis. More specifically, we consider a parameterized mitigation protocol that consists of the following modalities:

1) *High-risk sheltering*: criteria for high-risk individuals are established resulting in a ratio of the population (called $HRSratio$) requested to be in long-duration shelter. The high-risk individuals include those with underlying conditions that increase their risk of severe COVID-19 disease. Parameter: $HRSratio$.
2) *Social Distancing (SD)*: individuals are asked to follow social distancing guidelines. Parameters: (1) $NumOfCC$, defined as the number of close contacts per person per day without mitigation (as determined by ECT app - see below), and (2) $SDmitigationRatio$ defined as the ratio of reduction, due to mitigation, in $NumOfCC$. I.e., the number $MitigatedNumOfCC$ of close contacts per person per day reduced by $SDmitigationRatio$ is given by

$$MitigatedNumOfCC = NumOfCC * (1 - SDMitigationRatio)$$

3) *Personal Protection (PPE)*: individuals are requested to wear personal protection equipment (masks). Parameters: (1) $ProbStoE$ defined as the probability that a susceptible individual becomes exposed (E) in close contact with an infected (I) individual, when no PPE mitigation is used, and (2) $PPEMitigationRatio$ defined as the ratio of reducing $ProbStoE$ due to PPE mitigation. I.e., the mitigated probability $MitigatedProbStoE$ is given by

$$MitigatedProbStoE = ProbStoE * (1 - PPEMitigationRatio)$$

4) *Enhanced Contact Tracing (ECT)*: is a smart app randomly assigned to and used by a ratio of the population (called $ECTratio$) to identify close contact (e.g., within 6 ft for 15 minutes) of pairs of individuals. If an individual reports to the app that they have tested positive, then the ECT app alerts all individuals who have been in close contact with that individual, and requests certain actions, such as quarantine and testing, as described in the mitigation process protocol below. Parameters: (1)

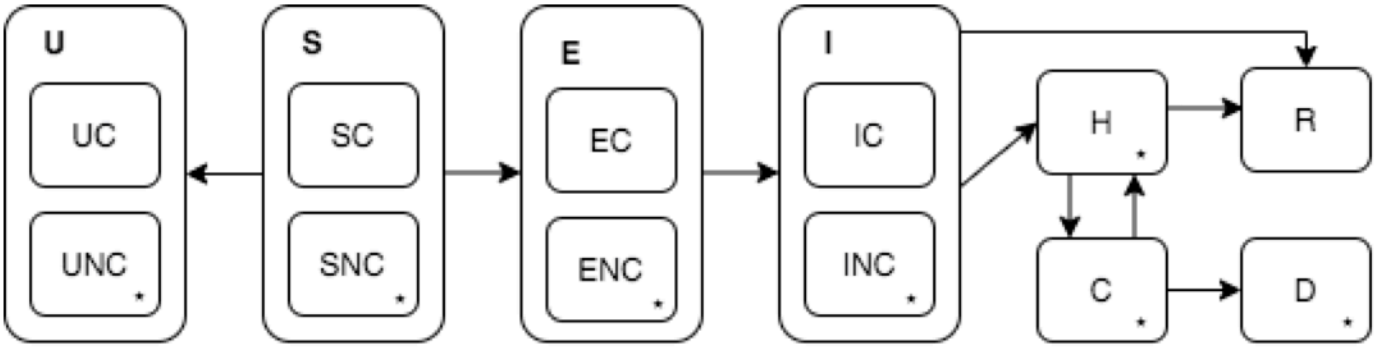

Fig. 1. Extended SUEIHCDR model Circulating C and Non-Circulating NC sub-compartments in U, S, E, I. Note: the asterisk denotes compartments and sub-compartments that are assumed to be non-circulating

$ECTratio$, and (2) $ECTcomplianceRatio$, defined as the probability that a random individual complies when requested to take an action by the ECT app.

5) *Symptom Reporting (SR)*: is a smart app randomly assigned to and used by a ratio of the population (called $SRratio$) to identify individuals with self-reported symptoms and/or exposures as early as possible to quarantine/isolate and possibly test them, as described in the mitigation process protocol below. Parameters: (1) $SRratio$, and (2) $SRcomplianceRatio$, defined as the probability that a random individual complies if requested to use the SR app.
6) *Testing*: individuals are requested to get a COVID-19 test as instructed by (1) ECT app, (2) SR app, or (3) random assignment of surveillance testing on a round-robin basis. This is part of the mitigation process protocol as described below (including its parameters.)

The mitigation process protocol, depicted in Figure 2, monitors all individuals on daily basis in terms of the following four $Cases = \{case1, case2, case3, case4\}$ that may arise:

- $case1$: individual marked by both ECT and SR apps.
- $case2$: individual marked by ECT app but not by SR app.
- $case3$: individual marked by SR app but not by ECT app.
- $case4$: individual not marked by either ECT or SR apps (asymptomatic testing).

If Case 1 or 3 are detected, the test is requested immediately, because an individual is symptomatic as per SR app. Once the test is administered and results are received and ready, the action depends on the positivity of the result. If positive, the individual is requested to move to the NC sub-compartment for at least Q days (e.g., 10 days.). If negative, the individual is not automatically kept in the C sub-compartment, because for some tests, the sensitivity is low, and some infected individuals may be negative. Instead, the protocol decides on whether to move the individual to NC with probability $P(NC|neg, case)$, which is a parameter given in the model input. Otherwise, the individual stays in the C sub-compartment. Note that, as a special case, if the probability $P(NC|neg, case) = 0$, no negatively tested individual will move to NC. As the other special case, if the probability $P(NC|neg, case) = 1$, all individuals will move to NC.

If Case 2 is detected, the test is not requested immediately but rather after the waiting period (e.g., 4 days of the incubation period) from the potential exposure as detected by the ECT app. This is because the exposed individual may still be in the incubation period, and so the test may be negative even though the individual will become infected (I) after the incubation period. Further actions taken in Case 2 are identical to that of Cases 1 & 3 up to the probability $P(NC|neg, case)$ which is case specific.

Otherwise, i.e., in Case 4, if the individual is not chosen for surveillance testing, they stay in the C sub-compartment. If the individual is chosen for testing, the test is requested immediately. Once the test is administered, and results are received and ready, the action depends on positivity of the result. If positive, the individual is moved to the NC sub-compartment. If negative, the individual stays in the C sub-compartment. Note that, in this case, the protocol does not randomly decides on moving the individual in NC, because there was no indication (i.e., marking by ECT and/or SR app) to do so.

III. Model Input and Output By Example

The input parameters required by the model, shown in Table I, fall into a few categories:

- *General Settings* including the duration of the time horizon, e.g., 150 day;
- *Initial Compartments* $s(0), u(0), e(0), \ldots, r(0)$, in terms of the number of individuals, for all compartments SUEIHCDR; and Non-Circulating *sub-compartments* $sNC(0), uNC(0), eNC(0), iNC(0)$ for the base sub-compartments SUEI.
- *Transition Ratios* that are time-independent for all transitions among the compartments as shown in Figure IV, except for the transition from U to S and from S to E, which change over the time horizon and are computed by the model.
- *Mitigation* parameters including social distancing and personal mitigation ratios.

A few of the input parameters include: time horizon, initial states, transition ratios, compliance ratio, mitigation ratios, and

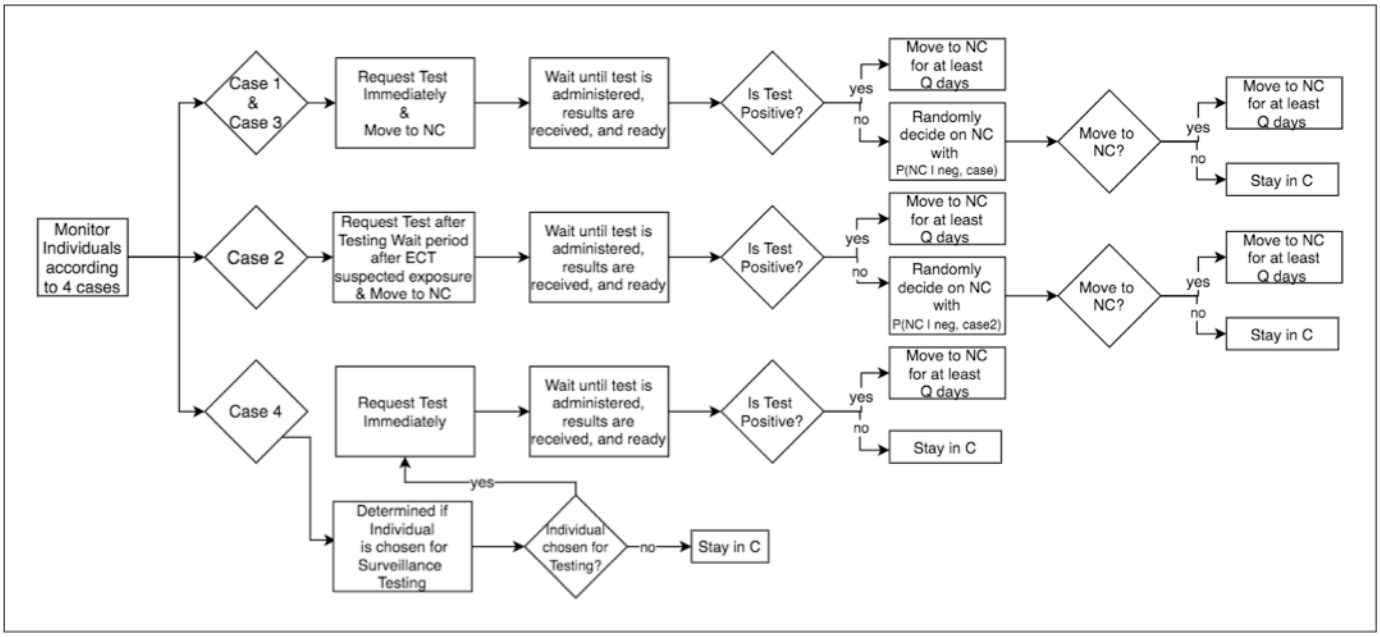

Fig. 2. Mitigation Process Protocol

testing details. An exhaustive list of all the inputs to the model, and the example values we are using found in this section. The inputs and their uses are described in Section IV.

Given the input parameters, the model output contains

1) The progression of the all the compartments and sub-compartments (C and NC) over the time horizon
2) Aggregated KPIs including (a) health outcomes, in terms of all compartments, both aggregated and over the time horizon, (b) the mitigation cost and its break-down (cost of tests, apps, quarantine), (c) productivity loss in terms of the total and percentage of non-circulating person-days.

So for the given example input, as seen in Table I, we can observe the progression of the compartments over the time horizon in Figures 3-7.

Figure 3 shows the progression of the Susceptible compartment (S, blue), and its two sub-compartments: Circulating (sC, green) and Non-Circulating (sNC, light blue) over the time horizon. At the start of the time horizon, there are 9360 individuals, and the number of individuals in S decreased over the time horizon, resulting in 9148 individuals on day 150. Similarly, the Circulating and Non-Circulating sub-compartments has with 9269 and 91 individuals, respectively, and at the end of the time horizon has 9065 and 82 individuals, respectively.

Figure 4 shows the progression of the Unsusceptible compartment (U, blue), and its two sub-compartments: Circulating (uC, green) and Non-Circulating (uNC, light blue) over the time horizon. There are 0 individuals in the Unsusceptible compartment initially, therefore, there are also 0 individuals in the sub-compartments as well. Over the time horizon, there is a gradual increase of individuals in all three. At the end of

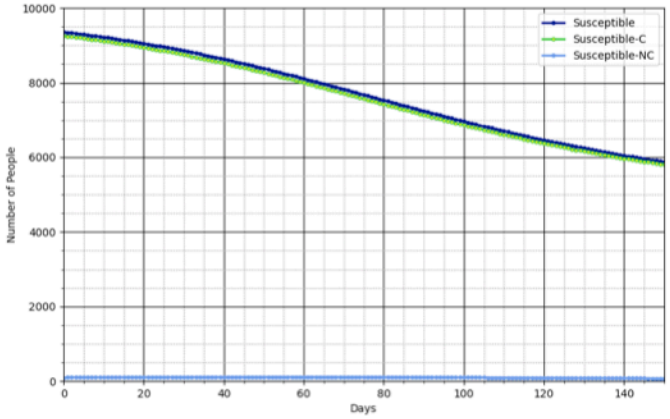

Fig. 3. Time Horizon graph for Susceptible compartment, and its 2 sub-compartments (sC and sNC)

the time horizon, there are 76, 75, and 1 individuals in the U, uC, and uNC compartments, respectively.

Figure 5 shows the progression of the Exposed compartment (E, blue), and its two sub-compartments: Circulating (eC, green) and Non-Circulating (eNC, light blue). All three of these compartments increase though the first half of the time horizon, but decrease thereafter. The compartments start off with 40, 18, and 22 individuals in the E, eC, and eNC, respectively. The peak number of exposed on any given day is 118 individuals. At the end of the time horizon, there are 62 individuals in E.

Figure 6 show the progression of the Infected compartment (I, blue), and its two sub-compartments: Circulating (iC, green) and Non-Circulating (iNC, light blue). Similarly to the Exposed compartments, all three infected compartments increase

Input Parameter	Example Value
General Settings	
t interval	day
Time Horizon	150
Epidemiology: Initial Compartments	
$pop(0)$	10000
$s(0)$	9360
$e(0)$	40
$i(0)$	100
$r(0)$	500
$u(0), h(0), c(0), d(0)$	0
$sNC(0)$	91
$uNC(0)$	1
$eNC(0)$	22
$iNC(0)$	81
Epidemiology: Transition Ratios	
$TR(e, i)$	0.25
$TR(i, r)$	9.99
$TR(h, r)$	0.08
$TR(h, c)$	0.02
$TR(c, h)$	0.08
$TR(c, d)$	0.02
Epidemiology: Durations	
$DurationE$	4
$DurationI$	10
Mitigation: High-Risk Sheltering	
$HRSratio$	0
Mitigation: Social Distancing	
$NumOfCC$	3.4
$SDMitigationRatio$	0.5
Mitigation: Personal Protection	
$ProbStoE$	0.705882353
$PPEMitigationRatio$	0.5
Mitigation: Enhanced Contact Tracing (ECT)	
$AppRatioECT$	0.9
$TrackingWindow$	10
$WaitECTtest$	4
Mitigation: Symptom Reporting (SR)	
$AppRatioSR$	1
$SympProbRatio$	0.5
$P(symp\|u)$	0.01
$P(symp\|s)$	0.01
$P(symp\|e)$	0.50
$P(symp\|i)$	0.7
Mitigation: Testing	
$AsympTestWin$	7
$NumTestsSR$	1
$NumTestsECT$	1
$WaitForRes$	1
$P(PosTest\|symp, u)$	0
$P(PosTest\|asymp, u)$	0
$P(PosTest\|symp, s)$	0
$P(PosTest\|asymp, s)$	0
$P(PosTest\|symp, e)$	0.475
$P(PosTest\|asymp, e)$	0.375
$P(PosTest\|symp, i)$	0.99
$P(PosTest\|asymp, i)$	0.75
$P(NC\|neg, case1)$	1
$P(NC\|neg, case2)$	0
$P(NC\|neg, case3)$	1
$P(NC\|neg, case4)$	0
Costs	
$PPEcost/day$	0
$ECTcost/person$	0
Misc	
$P(Compliance)$	0.9
$base$	0.0001

TABLE I
EXAMPLE INPUT FOR MODEL

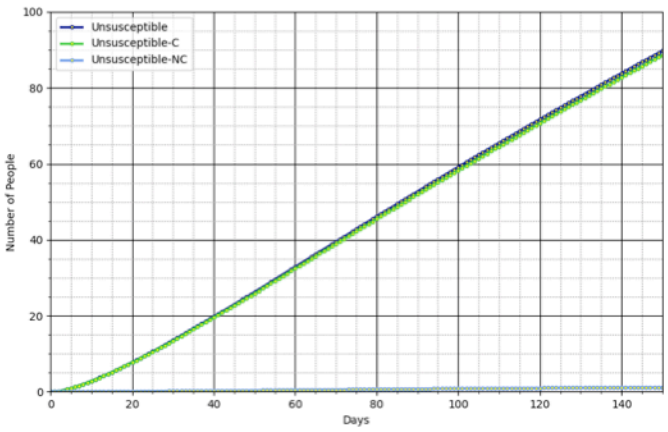

Fig. 4. Time Horizon graph for Unsusceptible compartment, and its 2 sub-compartments (uC and uNC)

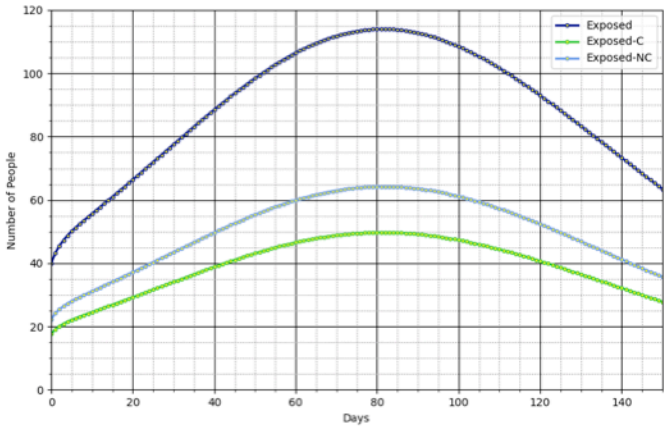

Fig. 5. Time Horizon graph for Exposed compartment, and its 2 sub-compartments (eC and eNC)

until day 90, and then gradually decrease. Initially there are 100, 19, 81 individuals in the I, iC, and iNC compartments, respectively. The peak of infections occurs on day 90 with 280 people infected.

Figure 7 shows the progression of the recovered compartment (R). There are 500 recovered individuals initially, and throughout the time horizon the number of individuals continue to increase, ultimately having 789 individuals in R at the end of the time horizon.

The model generates similar graphs for all other compartments and sub-compartments.

In addition to the progression of the compartments and sub-compartments over the time horizon, the model also computes various aggregated health, mitigation cost, and productivity outcomes. The aggregated health outcomes include, for each compartment $g \in \{E, I, H, C, D, R\}$, the total, peak, and days to peak, and the probability that a random individual will be in the g over the time horizon. The mitigation cost outcomes include total cost and its breakdown including the cost of tests, ECT and SR smart apps and isolation. Lastly, the productivity outcomes include *PersonDaysOut* - cumulative non-circulating

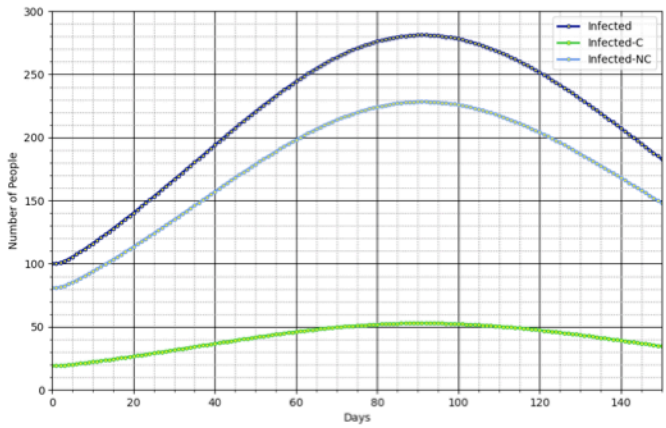

Fig. 6. Time Horizon graph for Infected compartment, and its 2 sub-compartments (iC and iNC)

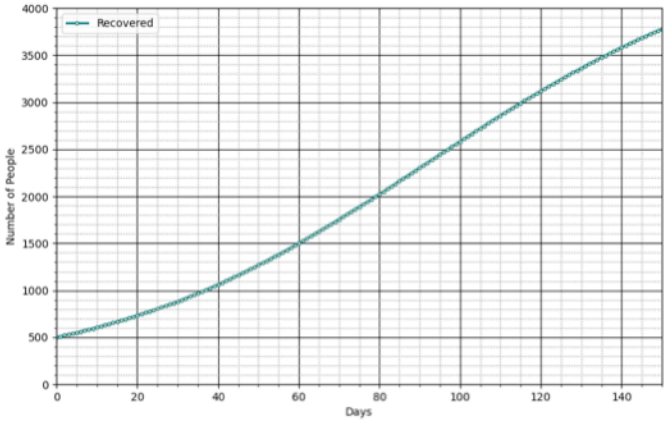

Fig. 7. Time Horizon graph for Recovered compartment

KPI	Definition	Example Value
Health Outcomes *(for* $g \in \{E, U, I, H, C, D, R\}$*)*		
$total(g)$	Total individuals in g over the time horizon	$total(i)$: 270.251
$peak(g)$	Maximum number of individuals in g	$peak(i)$: 100
$daysToPeak(g)$	Number of days to reach peak(g)	$daysToPeak(i)$: 0
$P(g)$	Probability that an random individual in the population was in g during time horizon	$P(e)$: 0.017
$Prob0Deaths$	Probability of 0 deaths during the time horizon	0.999
Mitigation Cost Outcomes		
Total Cost	Total cost of best protocol over time horizon	$10,306,661
Cost of Tests	Cost of testing over time horizon	$10,276,663
Cost of ECT apps	Cost of tracking apps over time horizon	$0
Cost of SR apps	Cost of symptom reporting apps over time horizon	$29,998
Cost of Isolation	Cost of quarantine (of all individuals) over time horizon	$0
Productivity Outcomes		
Person Days Out	Number of person-days out of Circulation over time horizon	11,612
Person Days Out (%)	Number of person-days out of Circulation over time horizon (%)	1.106%
Isolated Person Person Days Out	Number of person-days in isolation over time horizon	11,605
Isolated Person Person Days Out (%)	Number of person-days in isolation over time horizon (%)	1.105%
Cumulative NC days	Cumulative number of days individuals are Non-Circulating	11,612
Cumulative NC person-days (%)	(Cumulative NC days) / (time horizon * population)	1.106%

TABLE II
KEY PERFORMANCE INDICATORS (KPIS) BROKEN DOWN INTO 3 MAIN CATEGORIES: HEALTH, COST, AND PRODUCTIVITY

person-days. All the resulted KPIs for this example are shown in Figure II.

IV. DISCRETE DYNAMIC MODEL WITH MITIGATION

A. General Transitions

We consider a time horizon $T = \{0, \ldots, n\}$ of n days starting at time $t = 0$ (state before day one), and ending at time $t = n$ (state after day n). As seen in Figure 1, there are 8 base compartments $BC = \{U, S, E, I, H, C, D, R\}$, each having a number of subjects at each day t. For a compartment $g \in BC$, we denote by $g(t)$ the number of persons in g at time point t. The initial state $g(0)$ at time $t = 0$ for each compartment is given in the model input. Recursively, for $\forall g \in BC$, $\forall t \in \{1, \ldots, n\}$ we have

$$g(t) = g(t-1) + in(g,t) - out(g,t)$$

where $in(g,t)$ and $out(g,t)$ denote the number of individuals moving in to or out from, respectively, compartment g during day t, as described below.

Let $G = (BC, \mathcal{E})$ be a transition graph corresponding to the set BC of basic compartments in Figure 1, that is, $\mathcal{E} = \{(S,U), (S,E), (E,I), (I,H), (H,C), (H,R), (C,H), (C,D)\}$. For a transition edge $e = (g_1, g_2) \in \mathcal{E}$ we denote by $\delta(g_1, g_2, t)$ the number of individuals moving from compartment g_1 to compartment g_2 during day t. Therefore, $in(g,t)$ and $out(g,t)$ are expressed as follows:

$$in(g,t) = \sum_{g' \in \{g' | (g',g) \in \mathcal{E}\}} \delta(g', g, t)$$

$$out(g,t) = \sum_{g' \in \{g' | (g,g') \in \mathcal{E}\}} \delta(g, g', t)$$

In turn, for every transition edge $e = (g_1, g_2) \in \mathcal{E}$ and time point $t \in \{1, \ldots, n\}$

$$\delta(g_1, g_2, t) = TR(g_1, g_2, t) \times g_1$$

where $TR(g_1, g_2, t)$ denotes the transition ratio of individuals moving from compartment g_1 to compartment g_2 during the day t.

For every $e \in \mathcal{E}$, except for $e = (S,U)$ and $e = (S,E)$, the transition ratio $TR(g_1, g_2, t) = TR(g_1, g_2)$ is a time-

independent constant given in the model input. For $e = (S, U)$, we adopt the expression of $TR(g_1, g_2, t)$ from [11] as follows:

$$TR(S, U, t) = \frac{base * \log_2(t+1)}{\log_2(n+1)}$$

where $base$ is a constant given in the model input, and n is the length of the time horizon. The transition ratio for $e = (S, E)$ is more involved because we need to express its dependency on the mitigation protocol, as described in the next sections.

B. Transition Ratio $TR(S, E, t)$ per Mitigation Protocol

The transition ratio from susceptible S to exposed E compartment will depend on Circulating and Non-Circulating sub-compartments of the base compartments $\{U, S, E, I\}$, as depicted in Figure 1.

We also use the aggregate compartment Pop to denote the entire model population (i.e., from all basic compartments except for D and M.), and its Circulating and Non-Circulating sub-compartments $PopC$ and $PopNC$, respectively. In addition, we use the aggregate compartment Q which comprises of individuals from Non-Circulating sub-compartments of S, E and I. We extend the notation $g(t)$, the number of people in compartment g, to all $g \in BC \cup BCsub \cup \{Q, Pop, PopC, PopNC\}$.

Because only susceptible circulating (SC) population can get exposed to virus, for every $t = 1, \ldots, n$

$$TR(S, E, t) = P(SCtoE)(t) * \frac{SC(t-1)}{S(t-1)}$$

where $P(SCtoE)(t)$ is the probability for a random individual in SC to move to E during day t. In turn

$$P(SCtoE)(t) = 1-(1-MitigatedProbStoE)^{InfContacts(t)}$$

where $MitigatedProbStoE$ is the mitigated probability (see protocol PPE modality) that a random susceptible individual (i.e., in S) gets exposed (i.e., moves to E) given a single close contact with an infected (i.e., in I) individual. and $InfContacts(t)$ is the number of close contacts with circulating infected individuals (i.e., in IC) during day t. In turn

$$MitigatedProbStoE = ProbStoE * (1 - PPEMitigationRatio)$$

where $ProbStoE$ is the the unmitigated probability, and $PPEMitigationRatio$ is the mitigation parameter (see protocol PPE modality.) Both are parameters given in the model input. In turn $InfContacts(t)$ is

$$MitigatedNumOfCC * \frac{PopC(t-1)}{Pop(t-1)} * \frac{IC(t-1)}{PopC(t-1)}$$

where $MitigatedNumOfCC$ is the number of close contacts per person per day reduced by mitigation (i.e. social distancing, see protocol SD modality.).

$$MitigatedNumOfCC = NumOfCC * (1 - SDMitigationRatio)$$

where $NumOfCC$ is the number of close contacts per person per day, without mitigation, and $SDMitigationRatio$ is the mitigation ratio (as described in the protocol SD modality.). Both are parameters given in the model input.

Note that $InfContact$ is $MitigatedNumOfCC$ normalized for circulating population, and for the ratio of infected circulating population (IC) within the circulating population ($PopC$). Simplifying, we get

$$InfContacts(t) = MitigatedNumOfCC * \frac{IC(t-1)}{Pop(t-1)}$$

For every compartment $g \in \{U, S, E, I\}$ we need to express its sub-compartments gC and gNC. For $t = 0$, $gNC(0)$ is given as a parameter in the model input. For every $t = 1, \ldots, n$

$$gNC(t) = g(t) * P(NC|g)(t)$$

where $P(NC|g)(t)$ is the probability that a random individual in g is Non-Circulating (i.e., in gNC) at time point t. Finally, for every $t = 0, \ldots, n$

$$gC(t) = g(t) - gNC(t)$$

We discuss the computation of $P(NC|g)(t)$ in the following section.

C. Probability of Non-Circulating per Mitigation Protocol

For every compartment $g \in \{U, S, E, I\}$ and time point $t = 1, \ldots, n$

$$P(NC|g)(t) = HRSratio + (1-HRSratio)*P(NCrequest|g)(t)*P(Compliance)$$

where $HRSratio$ ratio of the total population that is high risk and requested to be in a long-term shelter, and $P(Compliance)$ is the ratio of the total population that is compliant with the protocol, both of which are parameters provided by the model input. $P(NCrequest|g)(t)$ is the probability that a random individual in compartment g will be asked (according to the mitigation protocol) to move to NC sub-compartment.

Let $P(ECT|g)(t)$ and $P(SR|g)(t)$ be probabilities that a random individual in compartment g was marked by ECT or SR, respectively, within $TrackingWindow$ ending at time point t, where $TrackingWindow$ is a parameter given in the model input. We discuss computation of these probabilities in the next Section IV-E. The computation of $P(NCrequest|g)(t)$ is dependent on four $Cases = \{case1, case2, case3, case4\}$ that may arise:

- $case1$: individual marked by both ECT and SR apps. Note that probability of this case is given by $P(case1|g)(t) = P(ECT|g)(t) * P(SR|g)(t)$.
- $case2$: individual marked by ECT app but not by SR app. Note that probability of this case is given by $P(case2|g)(t) = P(ECT|g)(t) * (1 - P(SR|g)(t))$.
- $case3$: individual marked by SR app but not by ECT app. Note that probability of this case is given by $P(case3|g)(t) = (1 - P(ECT|g)(t)) * P(SR|g)(t)$.

- $case4$: individual not marked by either ECT or SR apps (asymptomatic testing). Note that probability of this case is given by
$P(case4|g)(t) = (1-P(ECT|g)(t))*(1-P(SR|g)(t))$.

In turn, for every compartment $g \in \{U, S, E, I\}$ and time point $t = 1, \ldots, n$

$$P(NCrequest|g)(t) = \sum_{c \in Cases} P(NC|c, g)(t) * P(c|g)(t)$$

where $P(NC|c, g)(t)$ is the probability that an individual in g was asked to move to the NC sub-compartment given case $c \in Cases$ within $TrackingWindow$ of t, and; $P(c|g)(t)$ is the probability of case c occurring for individuals in g within $TrackingWindow$ of t. The computation of $P(NC|c, g)(t)$ for each case $c \in \{case1, case2, case3\}$ follows the mitigation protocol, i.e., it is

$$(1 - P(GRN|c, g)) + P(GRN|c, g) * P(NC|c, neg)$$

where

- $P(GRN|c, g)$ is the probability that all of the following holds for a random individual in g given case c: a test is administered, results are ready and negative; its computation is described in Section IV-D.
- $P(NC|c, neg)$ is the probability of keeping an individual in the NC sub-compartment given case c and negative test results. This probability is given as a parameter in the model input. Recall that the mitigation protocol allows this option in spite of the negative test result because of possible low sensitivity of the test.

Whereas, for $case4$ we have

$$P(NC|case4, g) = 1 - P(GRN|case4, g)$$

D. Computation of $P(GRN|c, g)$

The computation of $P(GRN|c, g)$ for cases $c \in \{case1, case2, case3, case4\}$ is case-dependant since testing in the mitigation protocol is case-dependant. For cases $c \in \{case1, case2, case3\}$

$$P(GRN|c, g) = P(GR|c) * P(NegTest|c, g)$$

where $P(GR|c)$ is the probability that a test is administered and ready given case c, and; $P(NegTest|c, g)$ is a the probability that a random individual in g has a negative test result given case c.

For $c \in \{case1, case3\}$, $P(NegTest|c, g)$ is

$$P(NegTest|c, g) = (1 - P(PosTest|symp, g))^{NumTestsSR}$$

where $P(PosTest|symp, g)$ is the probability of a random individual in compartment g has a positive test result given she was marked by SR app (within $TrackingWindow$), and; $NumTestsSR$ is the number of tests given to an individual marked symptomatic by the SR app; both values are parameters given in the model input. According to the mitigation protocol, in $case1$ and $case3$, an individual is tested immediately after becoming marked by the SR app. Thus, $P(GR|c)$ is estimated as

$$\frac{TrackingWindow - WaitForRes}{TrackingWindow}$$

where $WaitForRes$ is the number of days needed to receive test results after it is administered. Both values are constants given by the model input.

For $case2$, i.e., an individual was marked by the ECT app but not the SR app, we have

$$P(NegTest|case2, g) = (1 - P(PosTest|asymp, g))^{NumTestsECT}$$

where $P(PosTest|asymp, g)$ is the probability of an individual in compartment g that has a positive test result given they were not marked symptomatic by the SR app, and $NumTestsECT$ is the number of tests given to individual marked by the ECT app. Both values are constants given by the model input. In this case, according to the mitigation protocol, an individual must wait for $WaitECTtest$ days after potential exposure prior to taking a test. Thus $P(GR|case2)$ is estimated by

$$\frac{TrackingWindow - WaitECTtest - WaitForRes}{TrackingWindow}$$

For $case4$, according to the mitigation protocol, an individual may be selected for surveillance testing, as they have not been marked by either ECT or SR apps. The computation for $P(GRN|case4, g)$ depends on the compartment $g \in \{U, S, E, I\}$. If $g \in \{U, S\}$, we have

$$P(GRN|case4, g) = (1 - P(PosTest|asymp, g))^{AvgNumTests}$$

where $AvgNumTests$ is the average number of tests administered, which is estimated by

$$TrackingWindow/AsympTestWin$$

where $AsympTestWin$ is the number of days in which the entire asymptomatic population will be randomly tested in a round-robin manner, according to the mitigation protocol. All these parameters are given in the model input.

For compartment $g = E$

$$P(GRN|case4, g) = (1 - P(PosTest|asymp, g))^{AvgNumEtests}$$

where $AvgNumEtests$ is the average number of tests administered while the individual stays in the E compartment. It is estimated by

$$\frac{DurationE}{AsympTestWin}$$

where $DurationE$ is the number of days an individual stays in the compartment E (i.e, the incubation period of, say, 4 days.) It is a parameter provided in the model input.

Lastly, for $g = I$, to estimate $P(GRN|case4, g)$ note that an individual in this compartment at time point t may have been in I for a certain number of days, and in E prior to that, both within $TrackingWindow$ of time point t. Then

$$P(GRN|case4, g) = ProbNegI * ProbNegE$$

where $ProbNegI$, $ProbNegE$ are the probabilities that the individual tested negative (i.e., test given, result ready and negative) during the days she was in I and E, respectively. In turn

$$ProbNegI = (1 - P(PosTest|asymp, i))^{AvgNumItests}$$

where $AvgNumITests$ is the average number of tests during the I period, which is estimated by

$$\frac{DurationI/2 - WaitForRes}{AsympTestWin}$$

where $DurationI$ is a parameter given in the model input. Similarly

$$ProbNegE = (1 - P(PosTest|asymp, e))^{AvgNumEtests}$$

where $AvgNumEtests$ is the average number of tests during the E period, estimated earlier.

E. Computation of $P(ECT|g)(t)$ and $P(SR|g)(t)$

$P(ECT|g)(t)$is the the probability that a random individual in $g \in U, S, E, I$ is marked by the ECT app within $TrackingWindow$ of time point t.

$$P(ECT|g)(t) = 1-(1-P(CC|g)(t))^{NumOfCCInTrackingWin}$$

where $P(CC|g)(t)$ is the probability that a random individual in g will be marked by ECT in a single close contact (as defined by ECT, e.g., within 6 feet for at least 15 minutes) with another individual, and; $NumOfCCInTrackingWin$ is the number of close contacts with other individuals with the $TrackingWindow$, estimated as

$$MitigatedNumOfCC * TrackingWindow$$

To compute $P(CC|g)(t)$, note that marking an individual in a single close contact with another (1) both individuals must have ECT app, (2) the other individual must be infected, (3) know that she is infected, and (4) comply with reporting that to ECT. Therefore

$$P(CC|g)(t) = AppRatioECT^2 * RatioInfected(t-1) * P(KnownI|i) * P(Compliance)$$

where $AppRatioECT$ is ratio of population that has ECT apps, which is given as a model input; $RatioInfected(t-1) = IC(t-1)/popC(t-1)$ is estimated ratio of the circulating population that is infected (i.e., in compartment IC); $P(KnownI|I)$ is the probability of an individual in I knows that they are infected, and; $P(Compliance)$ is the ratio of the population that is compliant with the protocol, which is given by the model input.

$$P(KnowI|I) = P(KnowI|I, symp) * P(SR|I) + P(KnowI|I, asymp) * (1 - P(SR|I))$$

where $P(KnowI|I, symp)$ and $P(KnowI|I, asymp)$ are the probabilities that an individual in I knows they are infected given that they are symptomatic or asymptomatic, respectively; They are estimated as

$$P(KnowI|I, symp) = 1 - P(GRN|case3, I)$$

$$P(KnowI|I, asymp) = P(case2|I) * (1 - P(GRN|case2, I)) + P(case4|I) * (1 - P(GRN|case4, I))$$

where $P(GRN|c, g)$ is as defined in Section IV-D.

$P(SR|g)(t)$ is the the probability that a random individual in $g \in U, S, E, I$ is marked by the SR app within $TrackingWindow$ of time point t. It is estimated as

$$P(SR|g) = P(symp|g)* (AppRatioSR + (1 - SRratio) * SympProbRatio)$$

where $P(symp|g)$ is the probability that a random individual in g is symptomatic (as would be marked by SR app); $AppRatioSR$ is the ratio of the population that has SR apps, and; $SympProbRatio$ is ratio of $P(symp|g)$ for individuals who realize they are symptomatic without the use SR app. All of these are parameters given in the model input.

F. Aggregation Calculations

The aggregated KPI $total(g)$ for each compartment $g \in \{E, I, H, C, D, R\}$ is computed by aggregating $in(g, t)$ over the time horizon:

$$total(g) = g(0) + \sum_{t=1}^{n} in(g, t)$$

Similarly, the aggregated KPI $peak(g)$ for each compartment $g \in \{E, I, H, C, D, R\}$ is

$$peak(g) = \max_{t=0}^{n} g(t)$$

The number of days $daysToPeak(g)$ to reach the peak for eacg compartment $g \in \{E, I, H, C, D, R\}$ is

$$\min\{t \mid t \in \{0, \ldots, n\} \wedge g(t) = peak(g)\}$$

The probability $P(g)$ for a random individual to be in compartment $g \in \{E, I, H, C, D, R\}$ for at least one day over the time horizon is

$$P(g) = total(g)/pop(0)$$

where pop(0) is the initial population. The model also normalizes these probabilities in micromorts, i.e. the units of risk of 1 in 1 million. The $TotalCost = TestsCost + ECTcost + SRcost + IsolationCost$ and its components are computed

from cost per unit in the model input, and aggregated appropriately. The productivity outcomes are measured in terms of total number of person-days $PersonDaysOut$ that are not in circulating population, i.e.,

$$pop(0) * n - \sum_{t=1}^{n} (sC(t) + uC(t) + eC(t) + iC(t) + r(t))$$

They are also normalized to a percentage loss, i.e.,

$$PercentPersonDaysOut = PersonsDaysOut/(pop(0)*n)$$

Finally, $IsolatedPersonsDaysOut$ is the total of person-days in isolation, i.e., in the sub-compartment iNC, expressed as

$$\sum_{t=1}^{n} iNC(t)$$

as well is its percentage $PercentIsolatedPersonDays = IsolatedPersonDays/(pop(0) * n)$

V. Conclusion & Future Work

We developed the first model that, to the best of our knowledge, extends COVID-19 transmission dynamics with a comprehensive mitigation protocol, including interrelated modalities of (1) high-risk sheltering, (2) social distancing, (3) personal protection, (4) enhanced contact tracing (ECT) and symptoms reporting (SR) smart apps, and (6) testing instructed by ECT, SR and random surveillance assignment. With its ability to incorporate a complex mitigation protocol and determine predictive outcomes, this model can play a critical role in decision making in the development of strategic response efforts to the COVID-19 pandemic. However, additional research questions and opportunities for further development remain. These include the development of a more refined model with stochastic, as opposed to deterministic, description of disease dynamics. The challenge here is to be able to express the outcomes not through stochastic simulation, but in a closed analytic form, so that the model would be amenable to mathematical programming algorithms. This is important because mathematical programming algorithms significantly outperform simulation-based optimization algorithms in terms of optimality of results and computational complexity. Another interesting problem, beyond COVID-19 modeling, is the development of a modeling framework for various disease dynamics, incorporating an extensible model library for epidemiological and mitigation components. The technical challenge is to allow public health domain experts, who may not have mathematical or modeling background, to construct specific models using an intuitive high-level GUI which are interpreted by the system as a formal mathematical model used for mitigation recommendations. We believe that further use and development of this work can offer valuable insights for disease prevention and mitigation.

References

[1] Jinming Cao, Xia Jiang, Bin Zhao, et al. Mathematical modeling and epidemic prediction of covid-19 and its significance to epidemic prevention and control measures. *Journal of Biomedical Research & Innovation*, 1(1):1–19, 2020.

[2] Jun Chen, Lianlian Wu, Jun Zhang, Liang Zhang, Dexin Gong, Yilin Zhao, Qiuxiang Chen, Shulan Huang, Ming Yang, Xiao Yang, et al. Deep learning-based model for detecting 2019 novel coronavirus pneumonia on high-resolution computed tomography. *Scientific reports*, 10(1):1–11, 2020.

[3] Damian Clancy. Sir epidemic models with general infectious period distribution. *Statistics & Probability Letters*, 85:1–5, 2014.

[4] Ensheng Dong, Hongru Du, and Lauren Gardner. An interactive web-based dashboard to track covid-19 in real time. *The Lancet infectious diseases*, 20(5):533–534, 2020.

[5] Chris Dye and Nigel Gay. Modeling the sars epidemic. *Science*, 300(5627):1884–1885, 2003.

[6] Wayne M Getz, Richard Salter, Oliver Muellerklein, Hyun S Yoon, and Krti Tallam. Modeling epidemics: A primer and numerus model builder implementation. *Epidemics*, 25:9–19, 2018.

[7] Kayhan Ghafoor. Covid-19 pneumonia level detection using deep learning algorithm. 2020.

[8] Jinxing Guan, Yongyue Wei, Yang Zhao, and Feng Chen. Modeling the transmission dynamics of covid-19 epidemic: a systematic review. *Journal of Biomedical Research*, 34(6):422, 2020.

[9] Michelle L Holshue, Chas DeBolt, Scott Lindquist, Kathy H Lofy, John Wiesman, Hollianne Bruce, Christopher Spitters, Keith Ericson, Sara Wilkerson, Ahmet Tural, et al. First case of 2019 novel coronavirus in the united states. *New England Journal of Medicine*, 2020.

[10] Eghbal Hosseini, Kayhan Zrar Ghafoor, Ali Safaa Sadiq, Mohsen Guizani, and Ali Emrouznejad. Covid-19 optimizer algorithm, modeling and controlling of coronavirus distribution process. *IEEE Journal of Biomedical and Health Informatics*, 24(10):2765–2775, 2020.

[11] Deanna M Kennedy, Gustavo José Zambrano, Yiyu Wang, and Osmar Pinto Neto. Modeling the effects of intervention strategies on covid-19 transmission dynamics. *Journal of Clinical Virology*, 128:104440, 2020.

[12] Halgurd S Maghded, Kayhan Zrar Ghafoor, Ali Safaa Sadiq, Kevin Curran, Danda B Rawat, and Khaled Rabie. A novel ai-enabled framework to diagnose coronavirus covid-19 using smartphone embedded sensors: Design study. In *2020 IEEE 21st International Conference on Information Reuse and Integration for Data Science (IRI)*, pages 180–187. IEEE, 2020.

[13] Halgurd S Maghdid, Aras T Asaad, Kayhan Zrar Ghafoor, Ali Safaa Sadiq, and Muhammad Khurram Khan. Diagnosing covid-19 pneumonia from x-ray and ct images using deep learning and transfer learning algorithms. *arXiv preprint arXiv:2004.00038*, 2020.

[14] A David Paltiel, Amy Zheng, and Rochelle P Walensky. Assessment of sars-cov-2 screening strategies to permit the safe reopening of college campuses in the united states. *JAMA network open*, 3(7):e2016818–e2016818, 2020.

[15] Annelies Wilder-Smith, Calvin J Chiew, and Vernon J Lee. Can we contain the covid-19 outbreak with the same measures as for sars? *The lancet infectious diseases*, 20(5):e102–e107, 2020.

[16] Amin Yousefpour, Hadi Jahanshahi, and Stelios Bekiros. Optimal policies for control of the novel coronavirus disease (covid-19) outbreak. *Chaos, Solitons & Fractals*, 136:109883, 2020.